Ray tracing

CS425: Computer Graphics I

Khairi Reda

Overview

- Basic of ray tracing
- Object intersection
- Building a ray tracer

Ray tracing

- How to render realistic images?
  - Color
  - Materials
  - Rendering equation
- Alternative to rasterization: much more realistic images (at a cost!)

Ray tracing

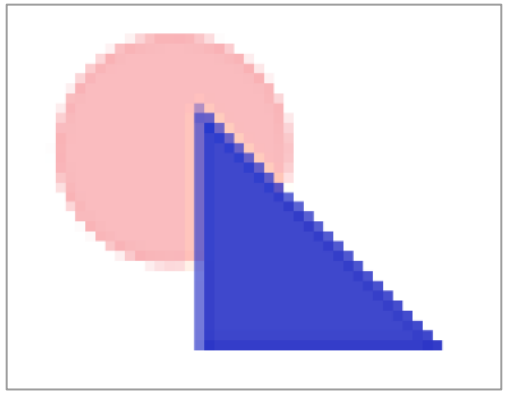
- Reverse of reality:
  - **Shoot rays through image plane.**
  - See what ray hits.
  - Secondary rays:
    - Reflections
    - Shadows

Ray tracing

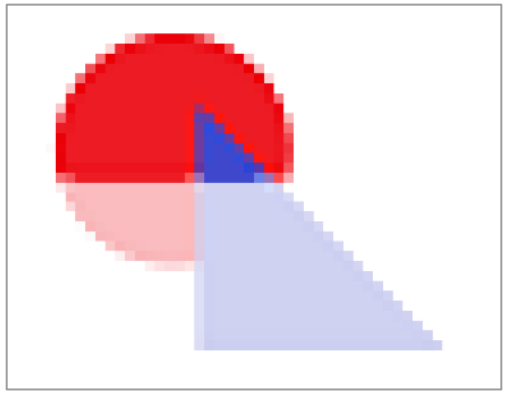
Ray tracing vs. rasterization

- Ray tracing and rasterization generate images in different ways.
- Basic difference: order in which samples are processed.

Rasterization



```
for each primitive:
    for each pixel:
        1. determine coverage
        2. evaluate color
```



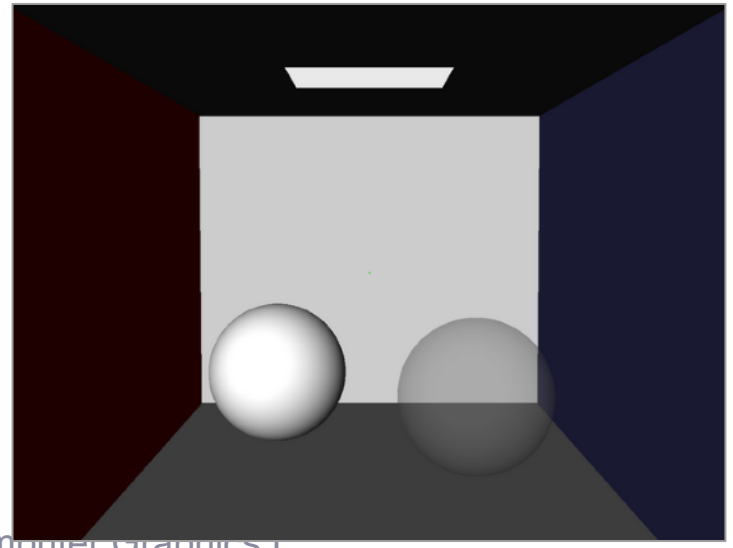
Ray tracing

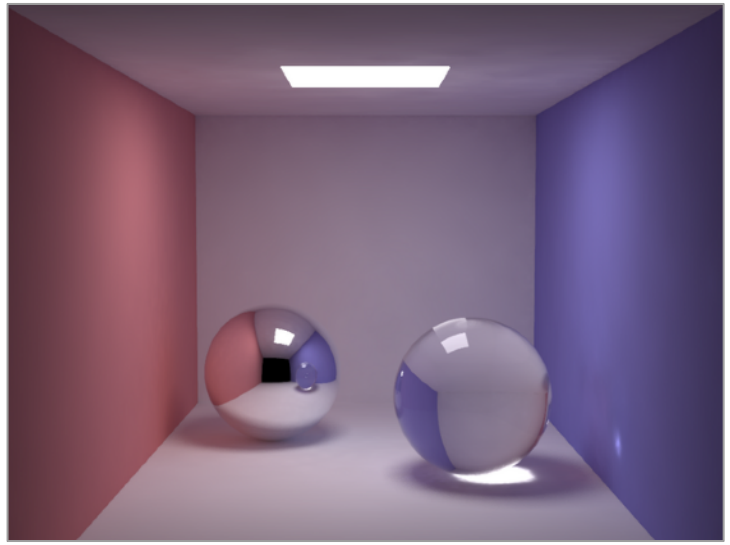
```
for each pixel:
    for each primitive:
        1. determine coverage
        2. evaluate color
```

Ray tracing vs. rasterization

- Illumination model:
  - Rasterization: local illumination – one primitive at a time.
  - Ray tracing: global illumination – one ray at a time (“easier” to determine global illumination effects).

Rasterization





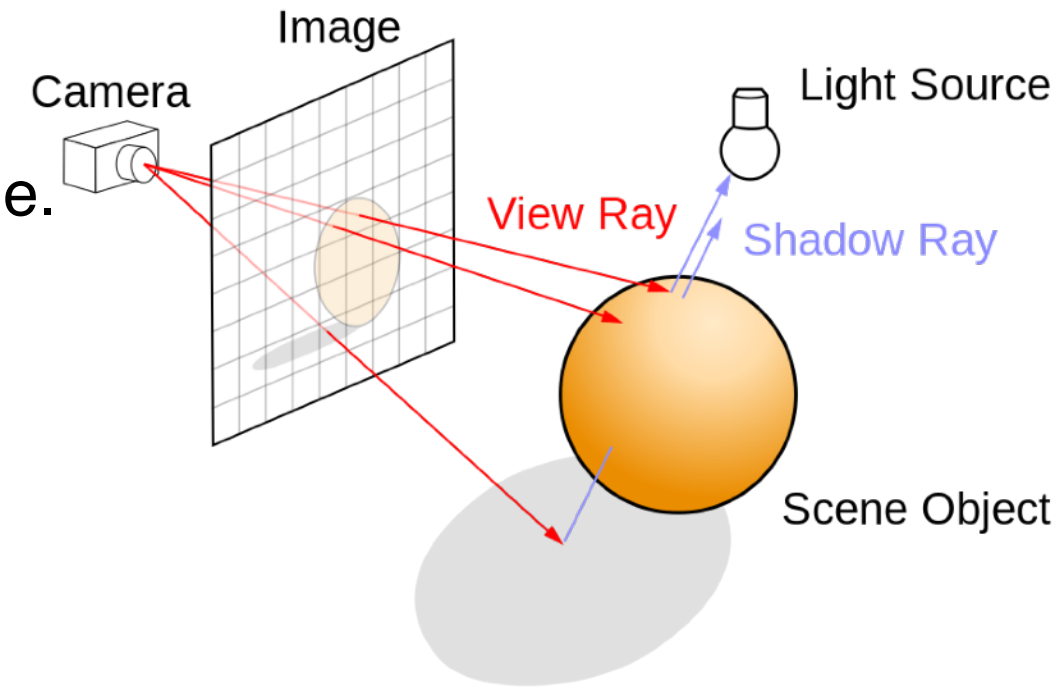
Ray tracing

Basic ray tracing

- Given a point source, follow rays of light as they traverse the scene.
  - Rays shot **from the eye through the pixel grid** into the scene.
  - Each ray: closest object intersection is found.
    - Shoot ray from intersection point to each light and find if it intersects with any object (if so, point in shadow).
- Account for reflection and transmission.

Basic ray tracing

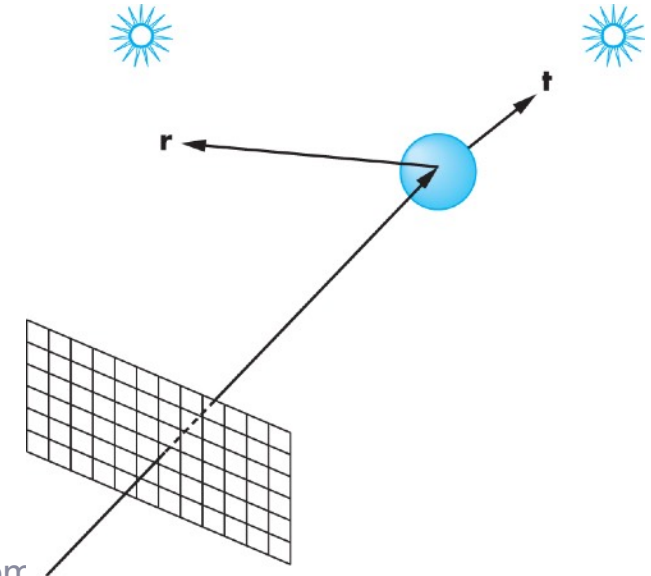
1. Create ray (one per pixel).
2. Intersect ray with objects in the scene.
3. Shade (compute color of the pixel)

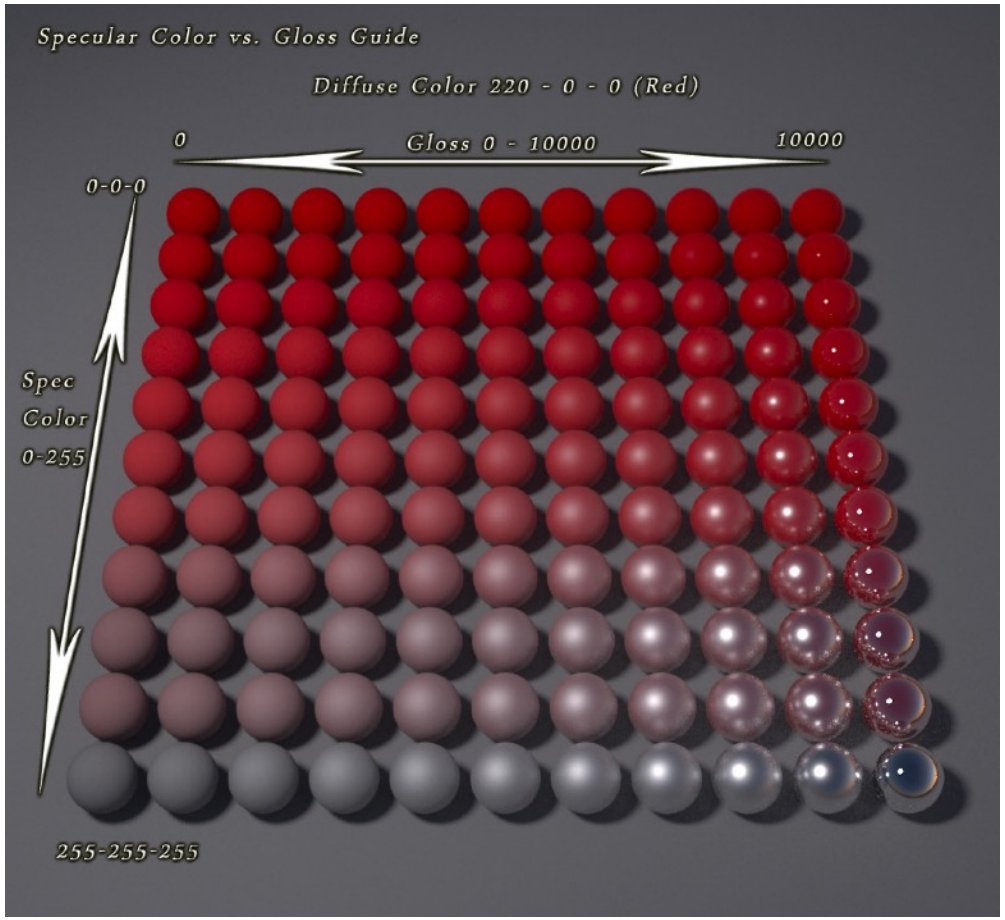


By Henrik - Own work, GFDL, https://commons.wikimedia.org/w/index.php?curid=3869326

Basic ray tracing

- Intersection ray-object will determine *secondary* rays, according to object material.



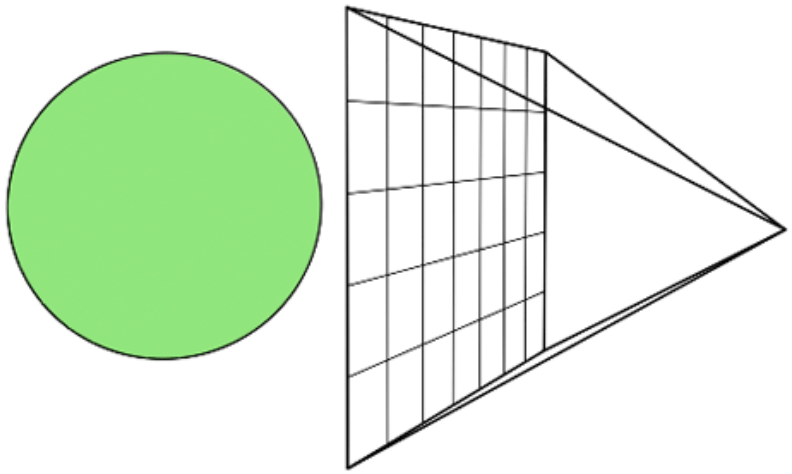


Constructing a ray

- Ray equation:

$$\mathbf{p}(t) = \mathbf{e} + t\mathbf{d}$$

- How to construct it?
- We have:
  - Camera position
  - Camera direction
  - Up vector
  - Viewport (width, height)
- We want:
  - Ray described by camera position and pixel center.



scratchapixel.com

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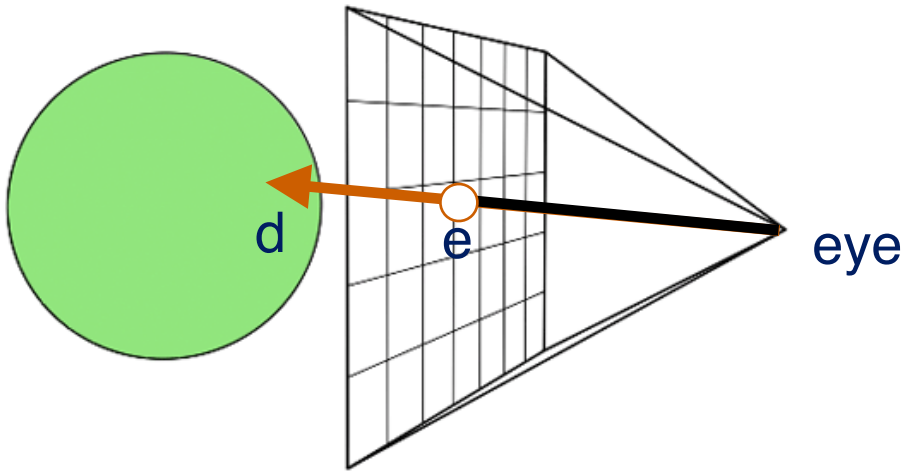


scratchapixel.com

Constructing a ray

- Ray equation:

$$\mathbf{p}(t) = \mathbf{e} + t\mathbf{d}$$

- How to construct it?

1. Transform pixel position to camera space:

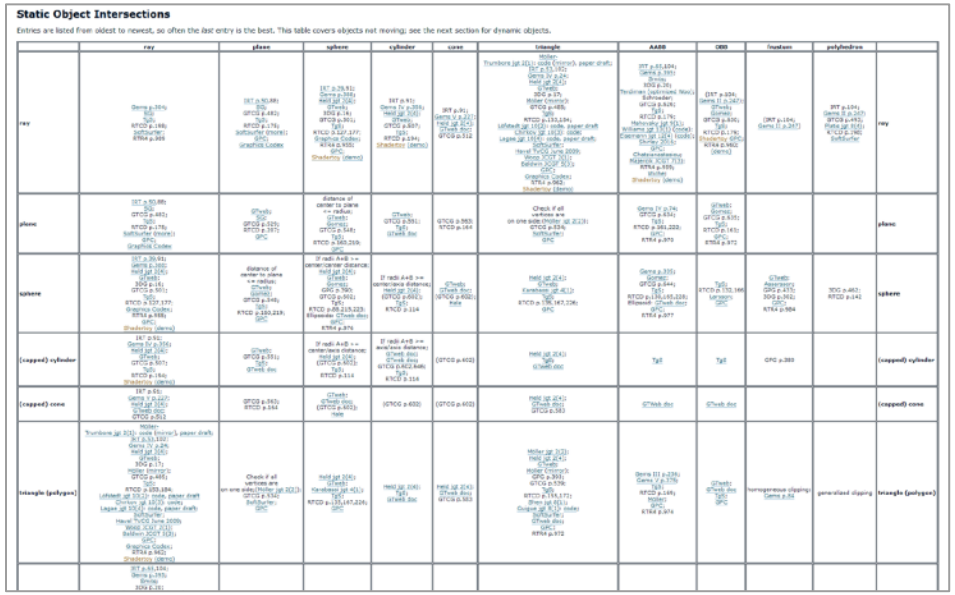
$$\mathbf{e} = ((2pixelScreen_x - 1) * AspectRatio * \tan\left(\frac{fov}{2}\right), \left(1 - 2pixelScreen_y\right) * \tan\left(\frac{fov}{2}\right), -1)$$

2. Ray direction:

$$\mathbf{d} = \|\mathbf{e} - \mathbf{eye}\|$$

Intersections

- Intersections are expensive operations.
- Great resource for operations:
  - http://www.realtimerendering.com/intersections.html
  - More than 100 types of intersections.
- We will focus on three:
  - Ray-plane intersection.
  - Ray-sphere intersection.
  - Ray-triangle intersection (we can approximate complex surfaces with triangles).



UIC COMPUTER SCIENCE

Ray-plane intersection

- Ray:

$$\mathbf{p}(t) = \mathbf{e} + t\mathbf{d}$$

- Plane (normal $\mathbf{n}$, point $\mathbf{p}_0$):

$$f(\mathbf{p}) = (\mathbf{p} - \mathbf{p}_0) \bullet \mathbf{n} = 0$$

- Intersection:

$$f(\mathbf{p}(t)) = 0$$

Ray-plane intersection

- Plugging ray equation into plane equation:

$$\left((\mathbf{e} + t\mathbf{d}) - \mathbf{p}_0\right) \bullet \mathbf{n} = 0$$

$$t = \frac{(\mathbf{p}_0 - \mathbf{e}) \bullet \mathbf{n}}{\mathbf{n} \bullet \mathbf{d}} \begin{cases} \mathbf{n} \bullet \mathbf{d} = 0 : \text{no intersection} \\ \mathbf{n} \bullet \mathbf{d} \neq 0 : \textit{one intersection} \end{cases}$$

Ray-sphere intersection

- Ray:

$$\mathbf{p}(t) = \mathbf{e} + t\mathbf{d}$$

- Sphere (radius R and center $\mathbf{c}$):

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = R^2$$

$$f(\mathbf{p}) = (\mathbf{p} - \mathbf{c}) \bullet (\mathbf{p} - \mathbf{c}) - R^2 = 0$$

- Intersection:

$$f(\mathbf{p}(t)) = 0$$

Ray-sphere intersection

- Plugging ray equation into sphere equation:

$$(\mathbf{e} + t\mathbf{d} - \mathbf{c}) \bullet (\mathbf{e} + t\mathbf{d} - \mathbf{c}) - R^2 = 0$$

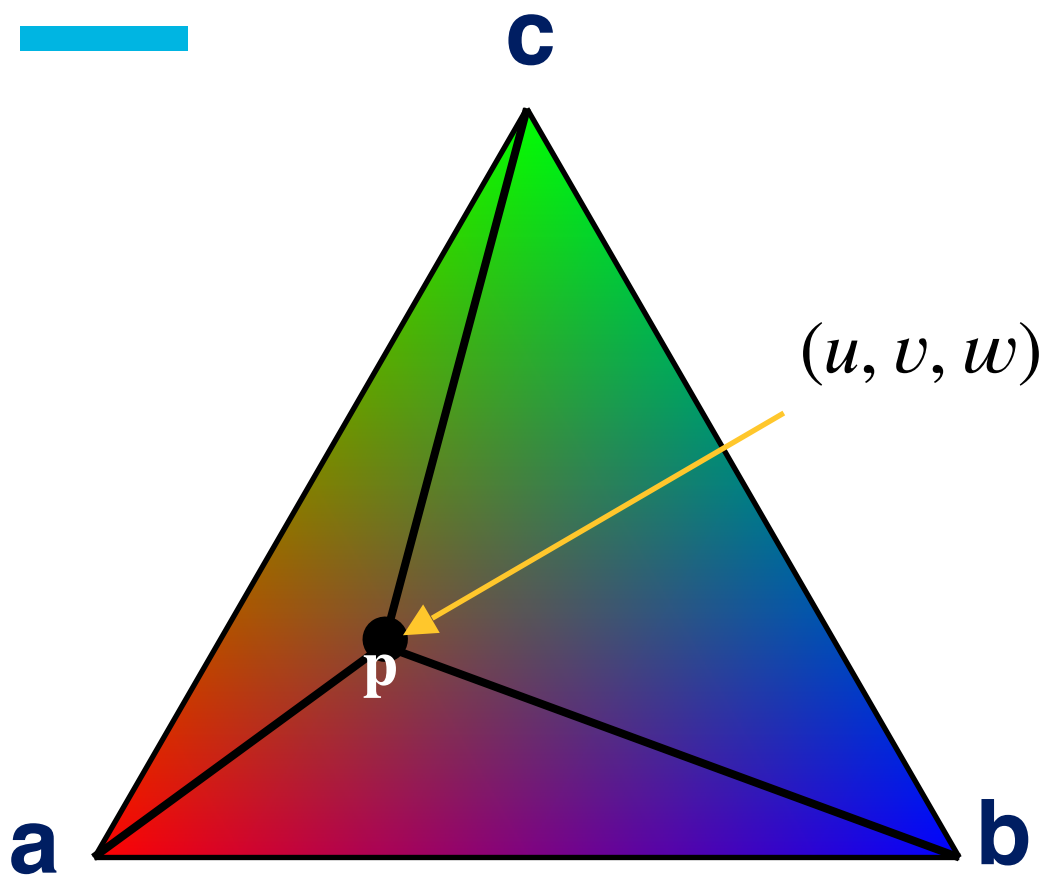
$$t\mathbf{d} \bullet t\mathbf{d} + 2t\mathbf{d}(\mathbf{e} - \mathbf{c}) + (\mathbf{e} - \mathbf{c}) \bullet (e - \mathbf{c}) - R^2 = 0$$

$$(\mathbf{d} \bullet \mathbf{d})t^2 + 2\mathbf{d} \bullet (\mathbf{e} - \mathbf{c})t + (\mathbf{e} - \mathbf{c}) \bullet (\mathbf{e} - \mathbf{c}) - R^2 = 0$$

$$At^2 + Bt + C = 0$$

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \begin{cases} B^2 - 4AC < 0 : no\ intersection \\ B^2 - 4AC = 0 : one\ intersection \\ B^2 - 4AC > 0 : two\ intersections \end{cases}$$

Barycentric coordinates



$$p = u\mathbf{a} + v\mathbf{b} + w\mathbf{c}$$
$$u + v + w = 1$$
$$u, v, w > 0 \quad \text{inside triangle}$$

$$u = 1 - v - w$$
$$\mathbf{f}(u, v) = \mathbf{a} + u(\mathbf{b} - \mathbf{a}) + v(\mathbf{c} - \mathbf{a})$$

Ray-triangle intersection

- Ray:

$$\mathbf{p}(t) = \mathbf{e} + t\mathbf{d}$$

- Triangle (vertices $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$):

$$\mathbf{f}(u, v) = \mathbf{a} + u(\mathbf{b} - \mathbf{a}) + v(\mathbf{c} - \mathbf{a})$$

- Intersection:

$$\mathbf{f}(u, v) = \mathbf{p}(t)$$

$$t > 0$$

$$0 \leq u, v$$

$$u + v \leq 1$$

Ray-triangle intersection

- Plugging ray equation into triangle:

$$\mathbf{a} + u(\mathbf{b} - \mathbf{a}) + v(\mathbf{c} - \mathbf{a}) = \mathbf{e} + t\mathbf{d}$$

$$\mathrm{u}(\mathbf{b} - \mathbf{a}) + v(\mathbf{c} - \mathbf{a}) - t\mathbf{d} = \mathbf{e} - \mathbf{a}$$

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

$$\begin{bmatrix} (\mathbf{b}-\mathbf{a})_\mathrm{x} & (\mathbf{c}-\mathbf{a})_\mathrm{x} & -d_\mathrm{x} \\ (\mathbf{b}-\mathbf{a})_\mathrm{y} & (\mathbf{c}-\mathbf{a})_\mathrm{y} & -d_\mathrm{y} \\ (\mathbf{b}-\mathbf{a})_\mathrm{z} & (\mathbf{c}-\mathbf{a})_\mathrm{z} & -d_\mathrm{z} \end{bmatrix} \begin{bmatrix} u \\ v \\ t \end{bmatrix} = \begin{bmatrix} (\mathbf{e}-\mathbf{a})_\mathrm{x} \\ (\mathbf{e}-\mathbf{a})_\mathrm{y} \\ (\mathbf{e}-\mathbf{a})_\mathrm{z} \end{bmatrix}$$

- How to solve the system?
  - Cramer's rule:

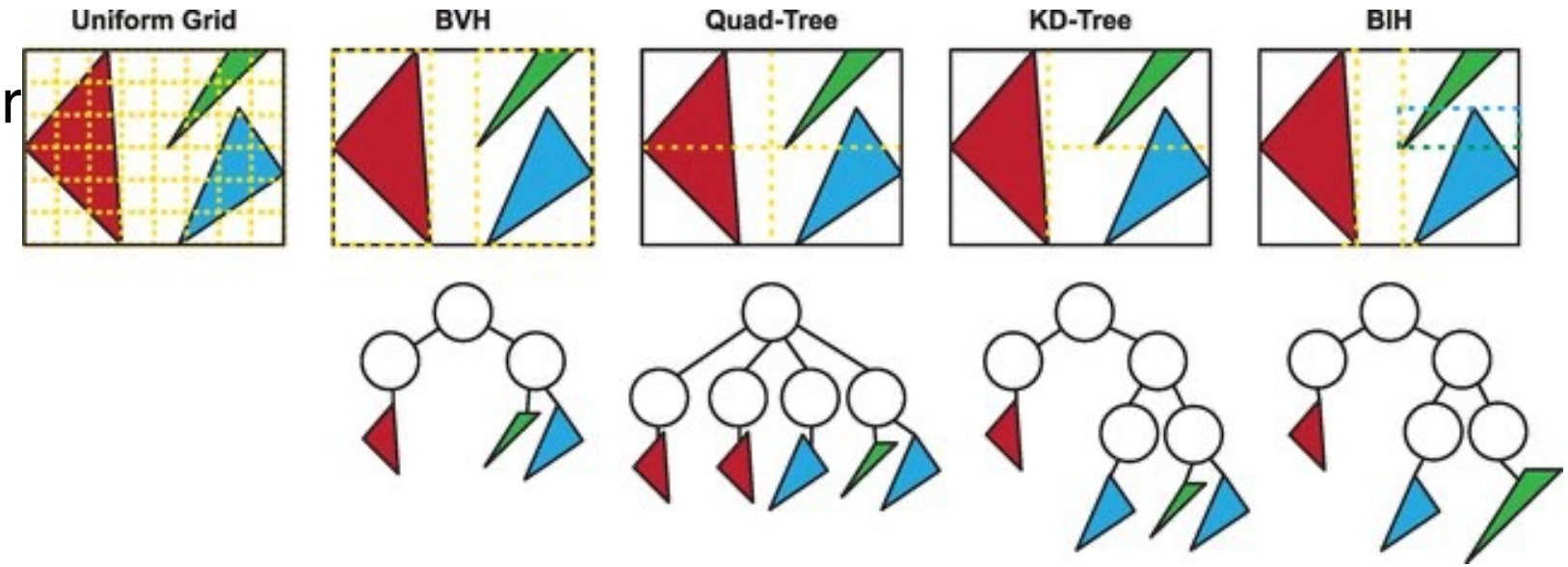
$x_i = \dfrac{\det(A_i)}{\det(A)}$, where $\mathbf{A}_i$ is the matrix $\mathbf{A}$ replacing the i-th column with vector $\mathbf{b}$

Multiple objects

- We must intersect each ray with **all** objects in the scene.
  - Complexity: O(n), where n is the number of primitives.
- To speed up computation, we can make use of spatial data structures to prune the number of collisions that we need to check.

Space-partitioning data structures

- Uniform grid
- Bounding volume hier (BHV)
- Quadtree / Octree
- KD-Tree



Review and Comparative Study of Ray Traversal Algorithms on a Modern GPU Architecture, Santos et al.

Space-partitioning data structures

- Uniform grid
- Bounding volume hierarchy (BHV)
- Quadtree / Octree
- KD-Tree

https://youtu.be/FLP4_CGs2Ng

Recursive ray tracing

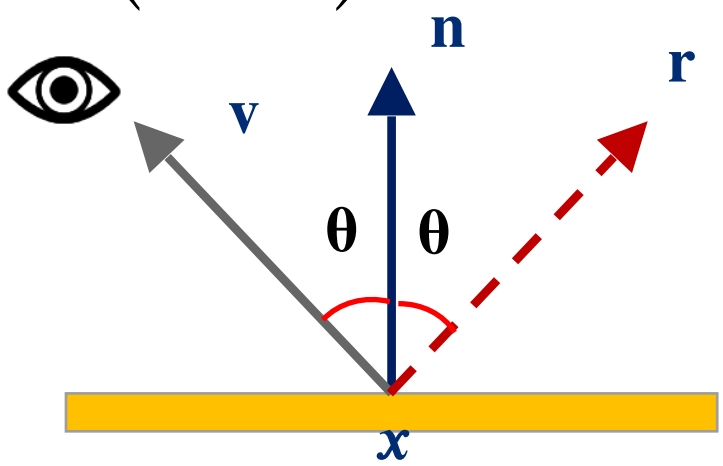
- Major steps of a ray tracer:

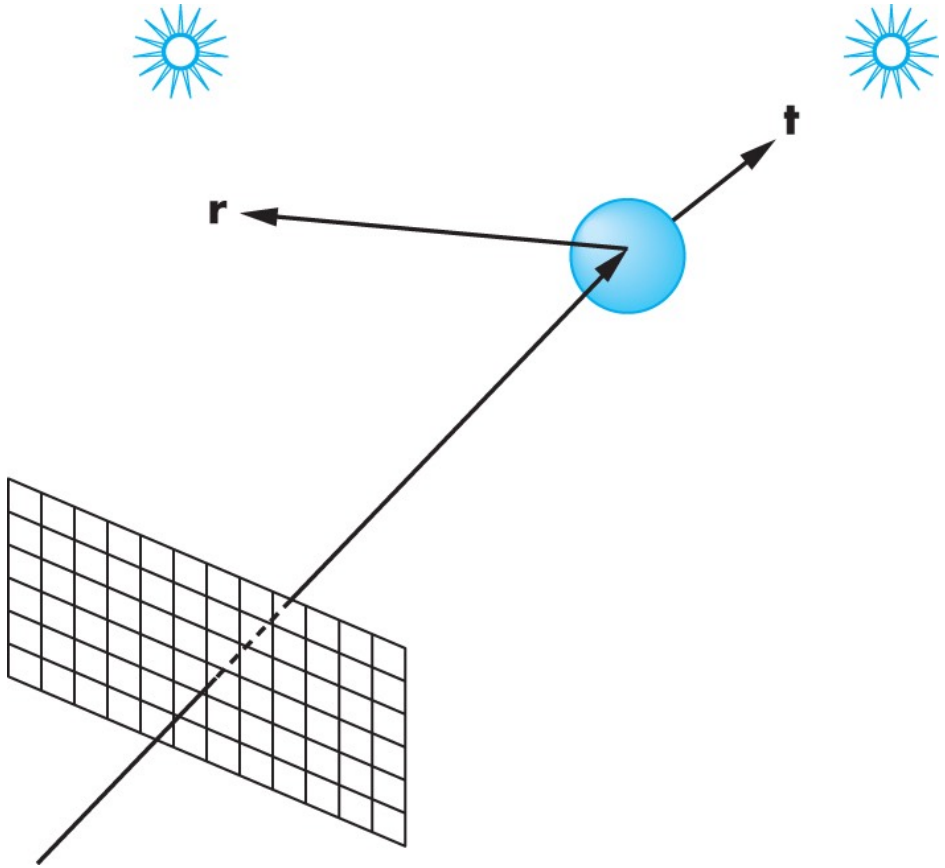
1. Determine intersection ray-objects.
2. Get intersected object material.
3. Based on material:
    1. Shade
    2. Reflect
    3. Transmit
4. When to stop? Define a maximum number of allowed recursions.

Reflections

- Reflected rays will follow the law of reflection:

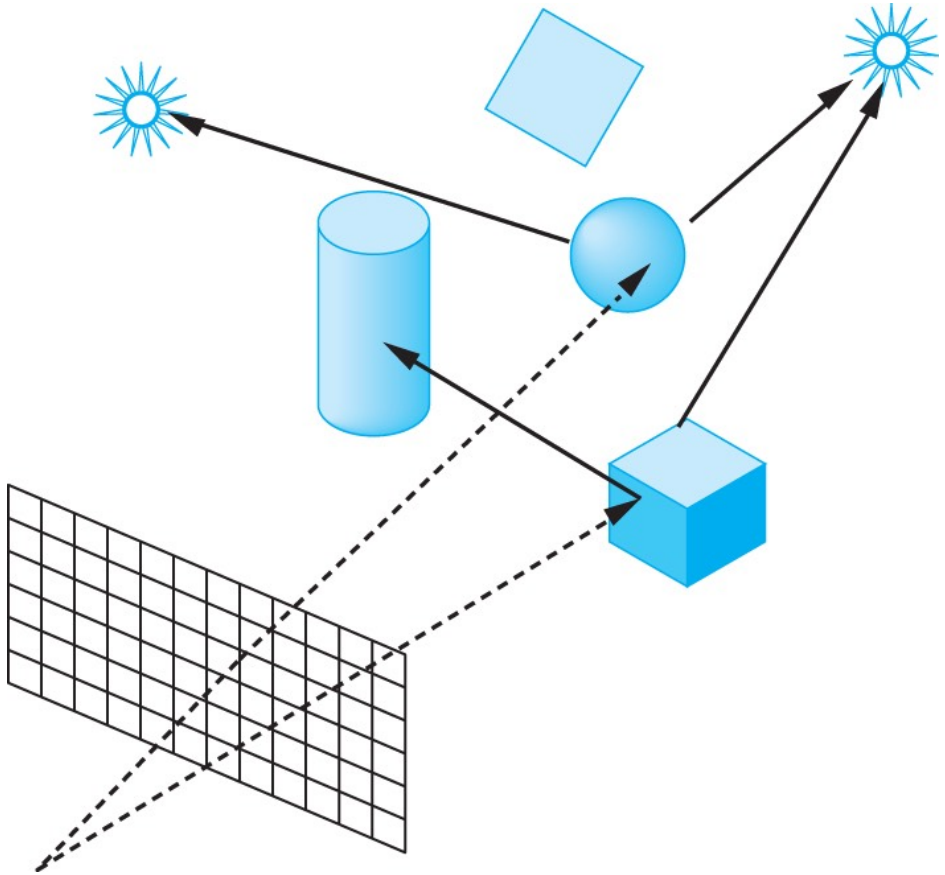
$$\hat{\mathbf{r}} = 2\left(\hat{\mathbf{v}} \bullet \hat{\mathbf{n}}\right)\hat{\mathbf{n}} - \hat{\mathbf{v}}$$





Shadows

- To check if a point is under shadow or not, cast a ray from each point to the light:
  - If it intersects something before reaching the light, it is in a shadow area.



Building a ray tracer

```
function render(scene) {
    var ctx = c.getContext("2d");
    var data = ctx.getImageData(0, 0, width, height);

    for(var x=0; x < width; x++) {
        for(var y=0; y < height; y++) {
            var color = trace(ray, scene, 0);
            // ...
        }
    }
    ctx.putImageData(data, 0, 0);
}
```

Render loop

Building a ray tracer

```
function trace(ray, scene, depth) {
    if(depth> MAX_DEPTH) return BACKGROUND_COLOR;
    var status = intersectObjects(ray, scene);
    if(status == no_intersection) return BACKGROUND_COLOR;

    var reflectedRay = // ...
    var reflectedColor = trace(reflectedRay, scene, depth+1);

    var refractedRay = // ...
    var refractedColor = trace(refractedRay, scene, depth+1);

    // ...

    return color;
}
```

Trace function

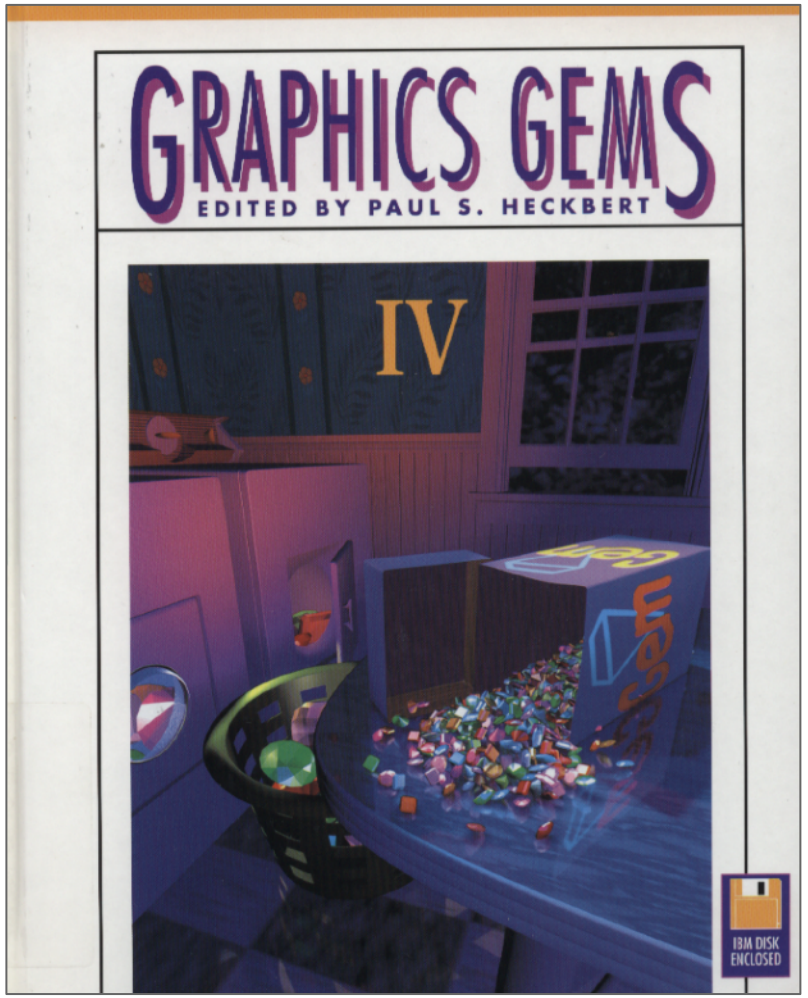
```
function intersectObjects(ray, scene) {
    for(var i=0; i < scene.objects.length; i++) {
        var object = scene.objects[i];
        var dist = rayTriangleIntersection(ray, object);
        // ...
    }
}
```

Intersect objects function

A minimal ray tracer

- Contest created by Paul S. Heckbert in 1987: "***write the shortest Whitted-style ray tracing program in C***".

PLACE	#TOKENS	AUTHOR	NOTES
		GENUINE ENTRIES	
1	916	Joe Cychosz, Purdue	compiler-dependent
1a	932	Joe Cychosz, Purdue	portable
2	956	Darwyn Peachey, Saskatchewan	portable
3	981	Michel Burgess, Montreal	portable
4	1003	Greg Ward, Berkeley	portable
		HONORABLE MENTIONS	
c1	10	Tony Apodaca, Pixar	cheater
c2	66	Greg Ward, Berkeley	cheater



A minimal ray tracer

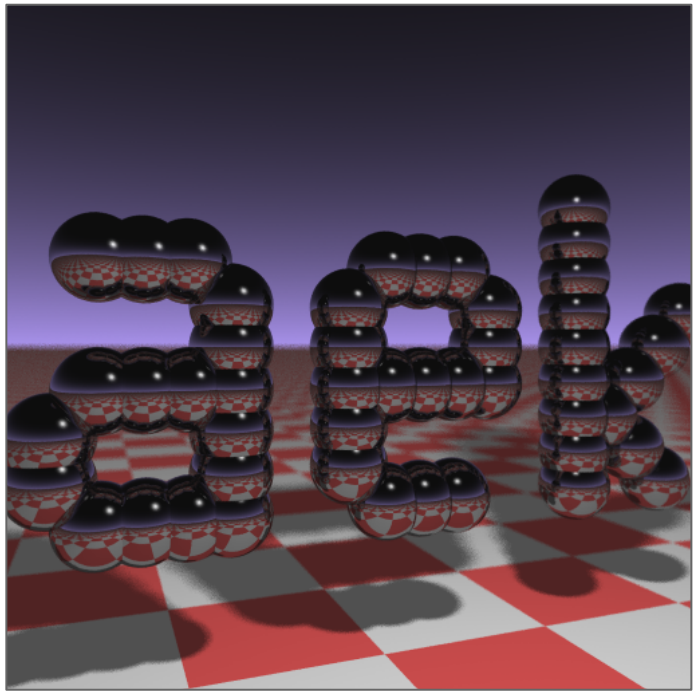
minray.card.c

```
typedef struct{double x,y,z}vec;vec U,black,amb={.02,.02,.02};struct sphere{
vec cen,color;double rad,kd,ks,kt,kl,ir}*s,*best,sph[]={0.,6.,.5,1.,1.,1.,.9,
.05,.2,.85,0.,1.7,-1.,8.,-.5,1.,.5,.2,1.,.7,.3,0.,.05,1.2,1.,8.,-.5,.1,.8,.8,
1.,.3,.7,0.,0.,1.2,3.,-6.,15.,1.,.8,1.,7.,0.,0.,0.,.6,1.5,-3.,-3.,12.,.8,1.,
1.,5.,0.,0.,0.,.5,1.5,};yx;double u,b,tmin,sqrt(),tan();double vdot(A,B)vec A
,B;{return A.x*B.x+A.y*B.y+A.z*B.z;}vec vcomb(a,A,B)double a;vec A,B;{B.x+=a*
A.x;B.y+=a*A.y;B.z+=a*A.z;return B;}vec vunit(A)vec A;{return vcomb(1./sqrt(
vdot(A,A)),A,black);}struct sphere*intersect(P,D)vec P,D;{best=0;tmin=1e30;s=
sph+5;while(s-->sph)b=vdot(D,U=vcomb(-1.,P,s->cen)),u=b*b-vdot(U,U)+s->rad*s
->rad,u=u>0?sqrt(u):1e31,u=b-u>1e-7?b-u:b+u,tmin=u>=1e-7&&u<tmin?best=s,u:
tmin;return best;}vec trace(level,P,D)vec P,D;{double d,eta,e;vec N,color;
struct sphere*s,*l;if(!level--)return black;if(s=intersect(P,D));else return
amb;color=amb;eta=s->ir;d= -vdot(D,N=vunit(vcomb(-1.,P=vcomb(tmin,D,P),s->cen
)));if(d<0)N=vcomb(-1.,N,black),eta=1/eta,d= -d;l=sph+5;while(l-->sph)if((e=l
->kl*vdot(N,U=vunit(vcomb(-1.,P,l->cen))))>0&&intersect(P,U)==l)color=vcomb(e
,l->color,color);U=s->color;color.x*=U.x;color.y*=U.y;color.z*=U.z;e=1-eta*
eta*(1-d*d);return vcomb(s->kt,e>0?trace(level,P,vcomb(eta,D,vcomb(eta*d-sqrt
(e),N,black))):black,vcomb(s->ks,trace(level,P,vcomb(2*d,N,D)),vcomb(s->kd,
color,vcomb(s->kl,U,black))));}main(){printf("%d %d\n",32,32);while(yx<32*32)
U.x=yx%32-32/2,U.z=32/2-yx++/32,U.y=32/2/tan(25/114.5915590261),U=vcomb(255.,
trace(3,black,vunit(U)),black),printf("%.0f %.0f %.0f\n",U);}/*minray!*/
```

A minimal ray tracer

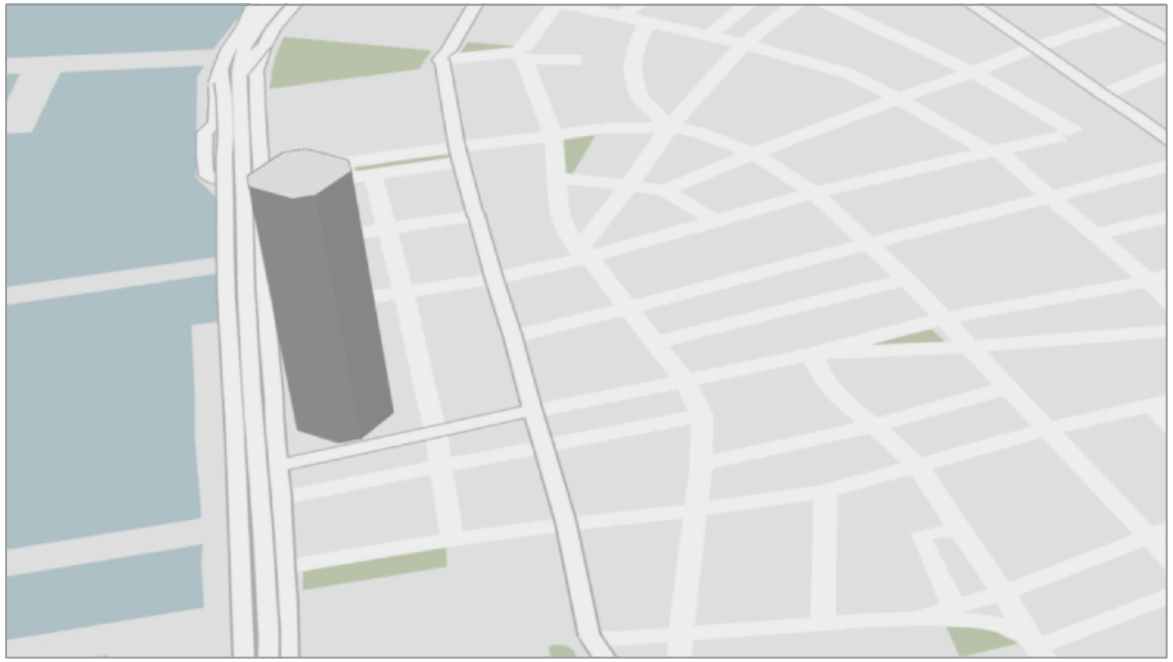
- Andrew Kensler's business card sized raytracer

```
#include <stdlib.h> // card > aek.ppm
#include <stdio.h>
#include <math.h>
typedef int i;typedef float f;struct v{ f x,y,z;v operator+(v r){return
v(x+r.x ,y+r.y,z+r.z);}v operator*(f r){return v(x*r,y*r,z*r);}f operator%(v r){return
x*r.x+y*r.y+z*r.z;}v(){}v operator^(v r ){return v(y*r.z-z*r.y,z*r.x-x*r.z,x*r. y-y*r.x);}
v(f a,f b,f c){x=a;y=b;z=c;}v operator!(){return*this*(1/sqrt(*this%* this));}};i
G[]={247570,280596,280600, 249748,18578,18577,231184,16,16};f R()
{ return(f)rand()/RAND_MAX;}i T(v o,v d,f &t,v&n){t=1e9;i m=0;f p=-o.z/d.z;if(.01
<p)t=p,n=v(0,0,1),m=1;for(i k=19;k--;) for(i j=9;j--;)if(G[j]&1<<k){v p=o+v(-k ,0,-
j-4);f b=p%d,c=p%p-1,q=b*b-c;if(q>0 ){f s=-b-sqrt(q);if(s<t&&s>.01)t=s,n=!
( p+d*t),m=2;}}return m;}v S(v o,v d){f t ;v n;i m=T(o,d,t,n);if(!m)return v(.7,
.6,1)*pow(1-d.z,4);v h=o+d*t,l=!(v(9+R( ),9+R(),16)+h*-1),r=d+n*(n%d*-2);f b=l%
n;if(b<0||T(h,l,t,n))b=0;f p=pow(l%r*(b >0),99);if(m&1){h=h*.2;return((i)(ceil( h.x)
+ceil(h.y))&1?v(3,1,1):v(3,3,3))*(b *.2+.1);}return v(p,p,p)+S(h,r)*.5;}i main()
{printf("P6 512 512 255 ");v g=!v (-6,-16,0),a=!(v(0,0,1)^g)*.002,b=!
(g^a )*.002,c=(a+b)*-256+g;for(i y=512;y--;) for(i x=512;x--;){v p(13,13,13);for(i r
=64;r--;){v t=a*(R()-.5)*99+b*(R()-.5)* 99;p=S(v(17,16,8)+t,!(t*-1+(a*(R()+x)+b
*(y+R())+c)*16))*3.5+p;}printf("%c%c%c" ,(i)p.x,(i)p.y,(i)p.z);}}
```



Shadow Accrual Maps

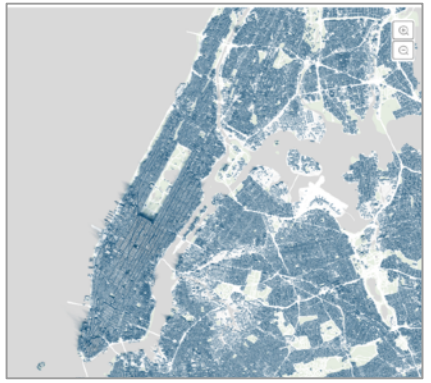
- Concrete example of ray tracing: shadow accumulation.
- “Shadow Accrual Maps: Efficient Accumulation of City-Scale Shadows Over Time”



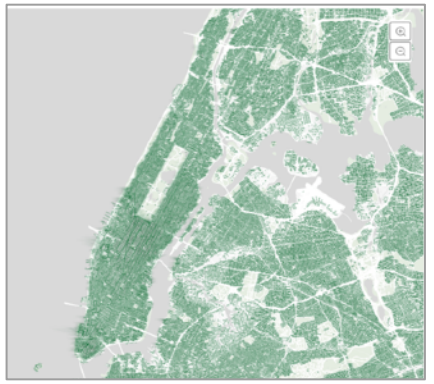
Shadow Accrual Maps: Efficient Accumulation of City-Scale Shadows over Time

Fabio Miranda, Harish Doraiswamy, Marcos Lage, Luc Wilson,
Mondrian Hsieh and Cláudio T. Silva

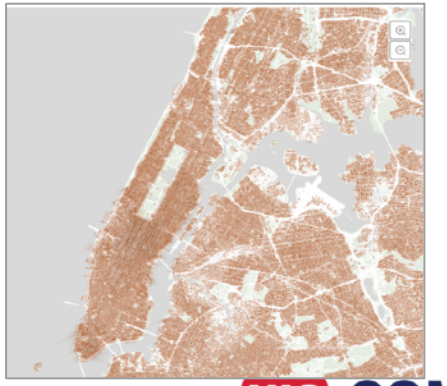
https://www.youtube.com/watch?v=LsZv23d1LyM



Winter



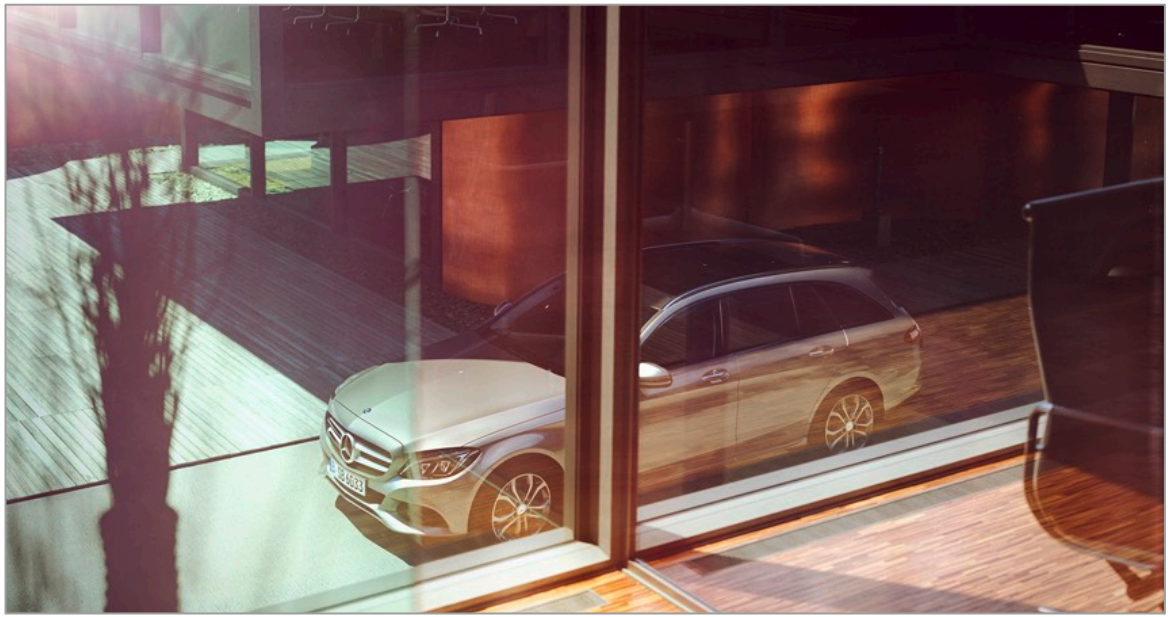
Spring / Fall



Summer

OptiX: modern ray tracer

- Ray tracing API by Nvidia.
- Applications beyond graphics: optical & acoustical design, radiation, etc.



OptiX: modern ray tracer

```
#include <optix.h>

optixContext = optix::Context::create();
optixContext->setRayTypeCount(1);
optixContext->setEntryPointCount(1);
optixContext->setStackSize(4640);
optixContext->setPrintEnabled(true);
optixContext->setExceptionEnabled(RT_EXCEPTION_ALL, true);



optixContext->setRayGenerationProgram(0, optixProgramGeneration);
optixContext->setExceptionProgram(0, optixProgramException);
optixContext->setMissProgram(0, optixProgramMiss);

optix::Acceleration acceleration = optixContext->createAcceleration("Lbvh");
acceleration->setTraverser("Bvh");
```

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```
#include <optix.h>

struct PerRayData_radiance
{
    float result;
    float importance;
    int depth;
};

rtDeclareVariable(optix::Ray, ray, rtCurrentRay, );

RT_PROGRAM void generation() {
    float4 pos = pointBuffer[launchIndex];
    float4 dir = directionBuffer[i];
    optix::Ray ray = optix::make_Ray(make_float3(pos), make_float3(dir), radianc
eRayType, sceneEpsilon, RT_DEFAULT_MAX);

    PerRayData_radiance prd;
    rtTrace(topObject, ray, prd);
    outputBuffer[launchIndex] += prd.result;
}
```

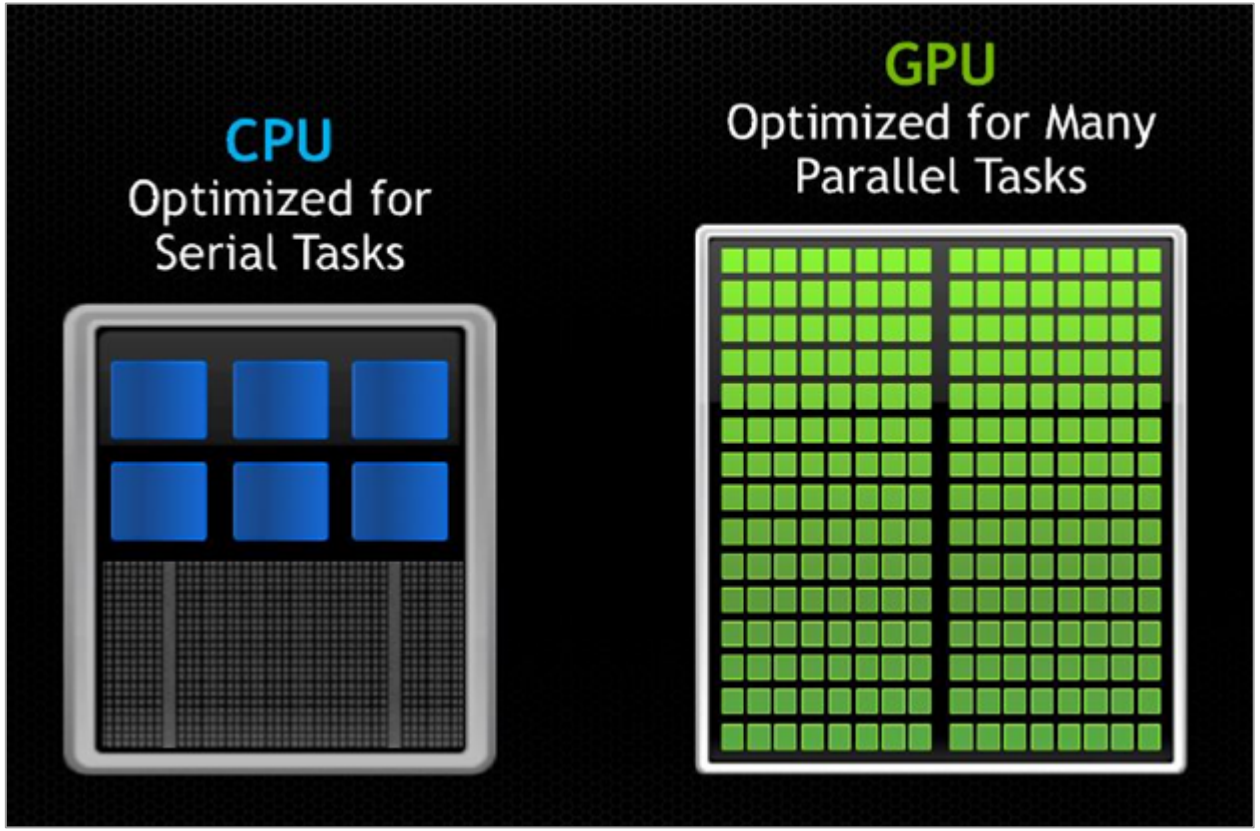
OptiX: modern ray tracer

```
RT_PROGRAM void miss() {
    prdRadiance.result = 0;
}

RT_PROGRAM void anyHit() {
    prdRadiance.result = 1;
    rtTerminateRay();
}

RT_PROGRAM void closestHit() {
    prdRadiance.result = 1;
    rtTerminateRay();
}
```

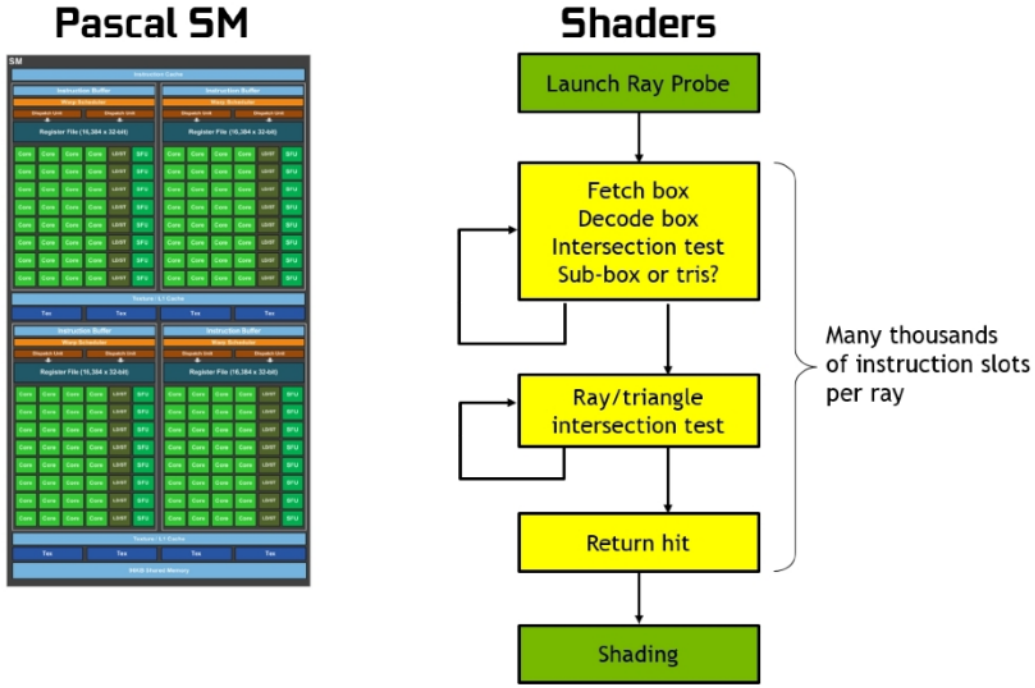
Modern GPUs


CPU
Optimized for
Serial Tasks
GPU
Optimized for Many
Parallel Tasks


SM
L1 Instruction Cache
L0 Instruction Cache
Warp Scheduler (32 thread/clk)
Dispatch Unit (32 thread/clk)
Register File (16,384 x 32-bit)
FP64
INT
FP32
TENSOR CORE
LD/ ST
SFU
128KB L1 Data Cache / Shared Memory
Tex

Ray-tracing cores

- RT cores: two specialized units for bounding box tests, and ray-triangle intersection tests.
- Frees up streaming multiprocessor (SM) from spending instruction slots per ray.



Pascal microarchitecture (pre Turing)

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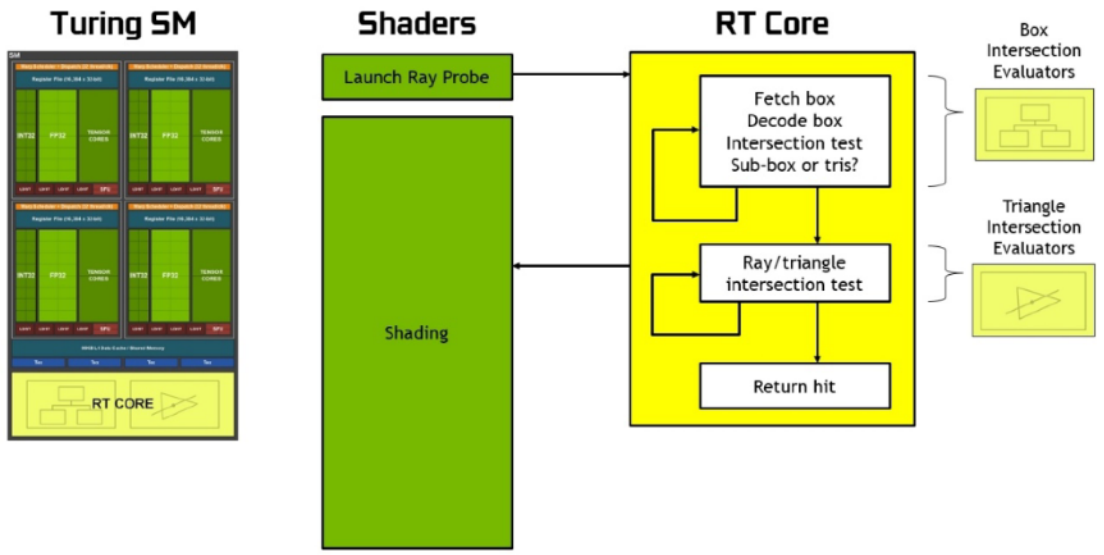

Turing microarchitecture

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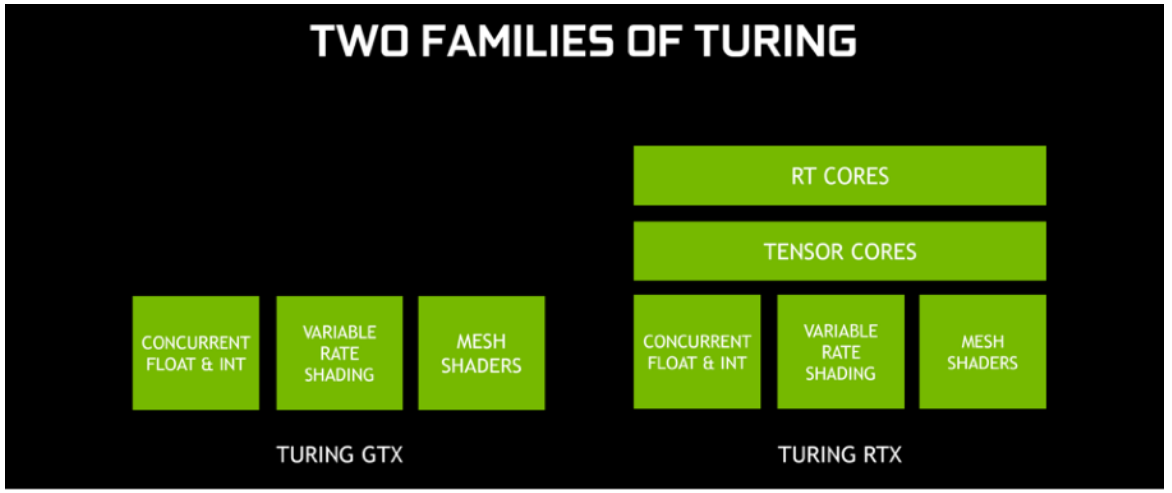

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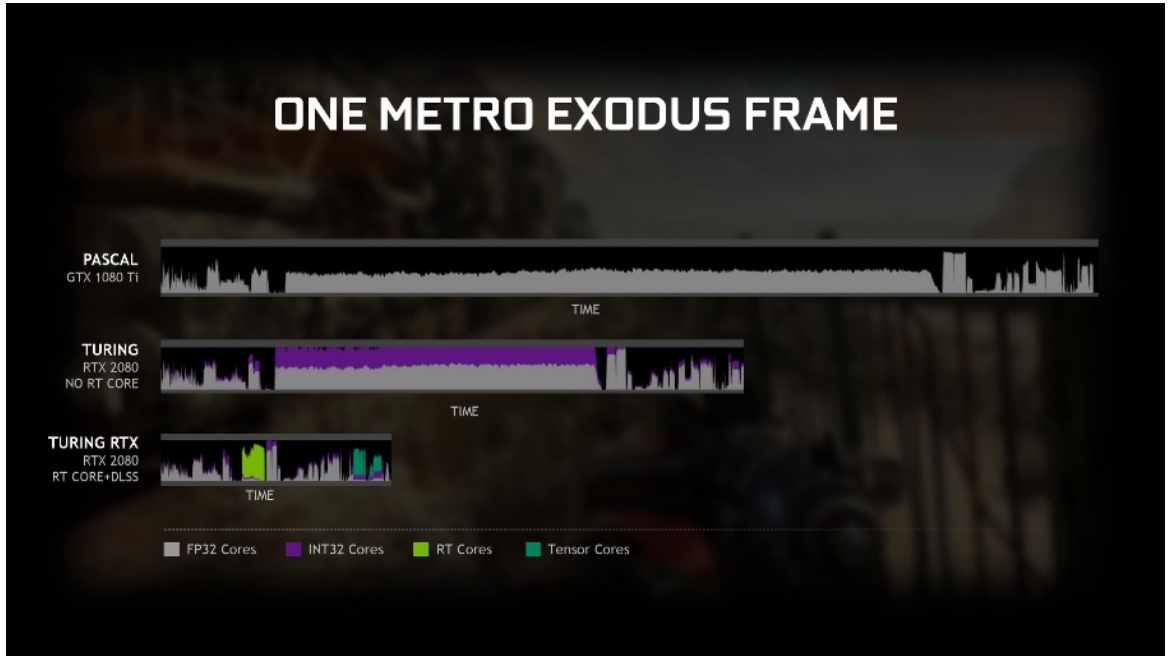
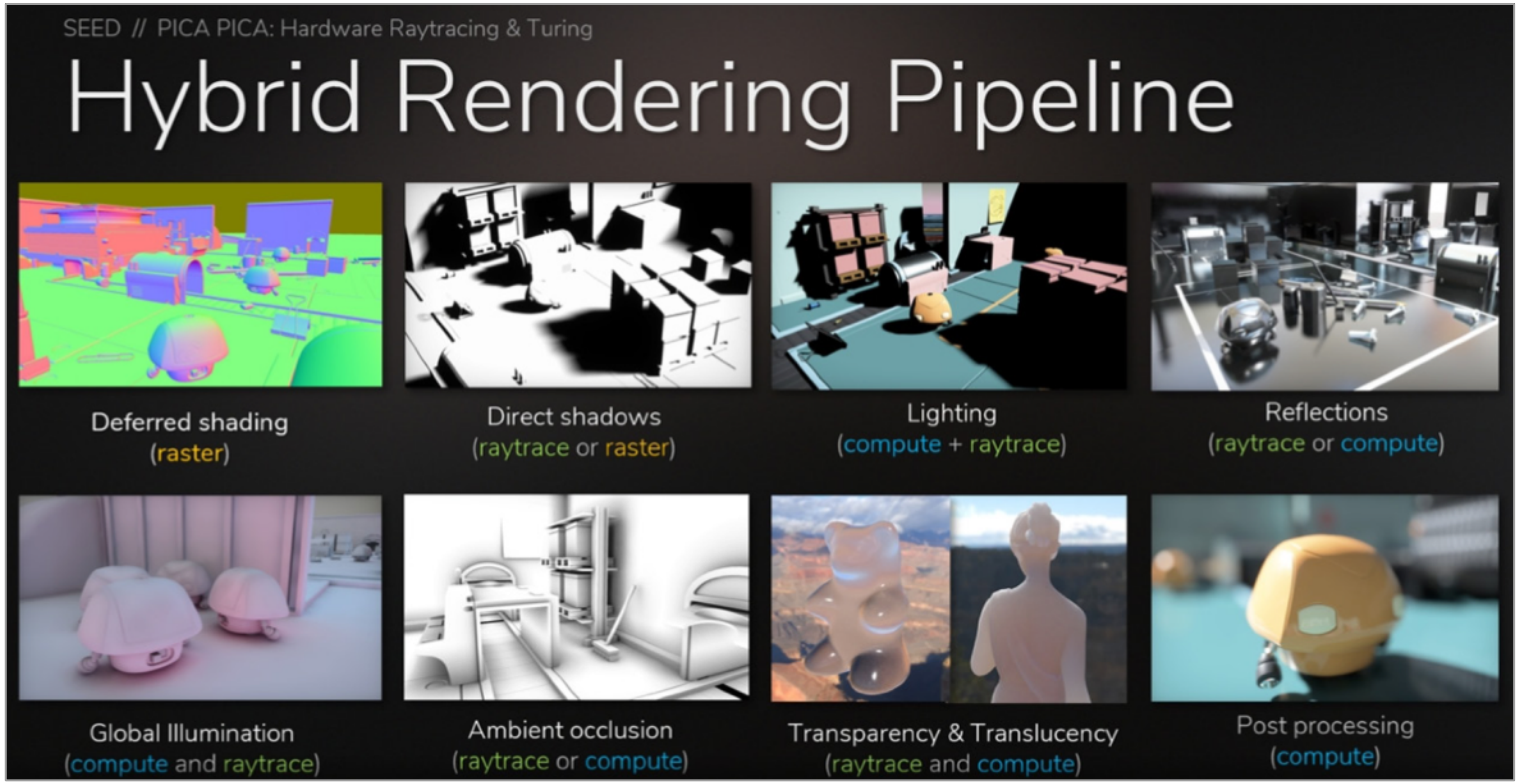


Hybrid rendering pipeline

- Rasterization and z-buffering are much faster at determining object visibility.
- Ray tracing can be used for secondary rays to correct reflections, refractions, and shadows.



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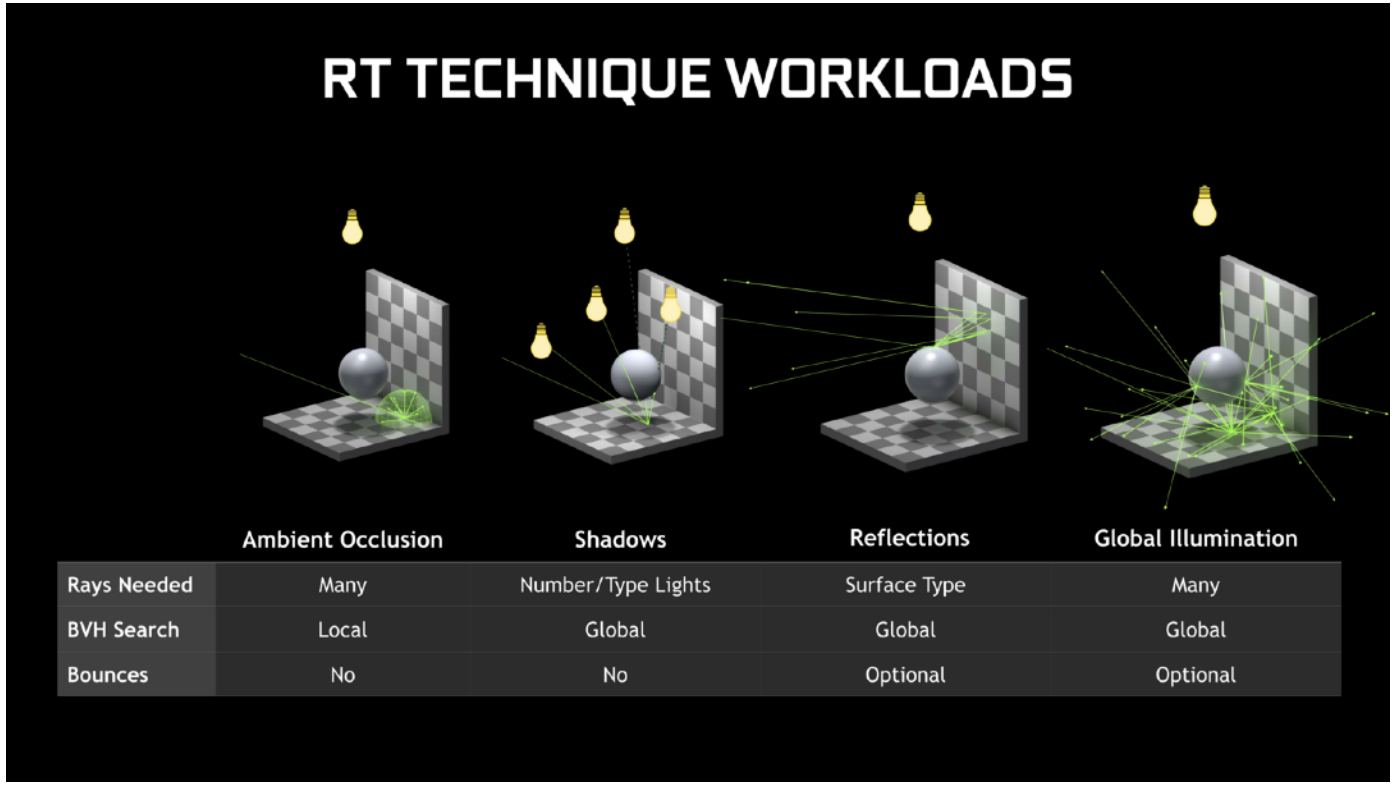

Hybrid rendering pipeline